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Water Surface Elevation due to Earthquake-Induced Sloshing

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ABSTRACT

The present paper provides an overview of approaches used to estimate the maximum water surface elevation in reservoirs due to earthquake-induced sloshing. These correspond to predictions based on the complete closed-form solution to the linearized boundary value problem, the corresponding solution for first mode sloshing only, and the early solution of Housner. These three approaches are compared and an example application is provided. Finally, a brief indication is also provided of various additional factors which may require consideration, including the treatment of reservoirs with complex planforms; the influence of the direction of base motion; and the estimation of hydrodynamic damping.

INTRODUCTION

The prediction of the hydrodynamic loads and water surface elevations in water-filled reservoirs and tanks is an important requirement in structural design. The traditional approach to estimating these has been outlined, for example, by Housner (U.S. Atomic Energy Commission, 1963) and in the AWWA (1984) and API (1993) standards. In general, such estimates are based on the assumption that the influence of the second and higher sloshing modes is negligible. Loads are generally estimated by the use of an impulsive, or high frequency, effective fluid mass which accelerates with the container, together with an additional effective fluid mass which undergoes resonant motions at the lowest natural frequency of sloshing.

In the case of elevation estimates, there is no term analogous to the impulsive force component, which is usually dominant, and therefore the effect of second and higher sloshing modes may not be negligible - particularly for large reservoirs for which the spectral acceleration may be quite low at the first mode natural period. Thus, several sloshing modes should be considered simultaneously. This may be done by applying an available closed-form solution of the corresponding linearized boundary value problem for a harmonic base motion extended to the case of earthquake-induced base motions.

The present paper provides an overview of approaches used to estimate the maximum water surface elevation due to earthquake-induced sloshing. These correspond to predictions based on the complete closed-form solution, the corresponding solution for first mode sloshing only, and the early solution of Housner. These three approaches are compared. It is shown that the reliable estimation of the maximum surface elevation in a large reservoir requires the influence of higher sloshing modes to be taken into account. And, as expected, Housner's predictions correspond closely to those based on the closed-form solution for first mode excitation, but give predictions which are slightly higher. A brief indication is also provided of various additional factors which may require consideration, including the treatment of reservoirs with complex planforms; the influence of the direction of base motion; and the estimation of hydrodynamic damping.

HARMONIC MOTION

Figure 1 provides a definition sketch of a rectangular reservoir: a denotes the half length of the reservoir, b denotes the width, and h denotes the water depth. Initially, the closed-form solution for the water surface elevation is summarized, and this is subsequently extended to earthquake motions. The solution is obtained on the basis of assumptions that the reservoir is rigid, the fluid is incompressible and inviscid, and the oscillation amplitude is small (such that the corresponding boundary value problem is linearized). The solution provides a description of the corresponding fluid motion, and thereby provides an expression for the maximum water surface elevation. Although the solution was initially developed for the case of no energy dissipation, it is possible to extend the solution to the case of energy dissipation corresponding to a specified damping coefficient, by assuming this to occur at the free surface (e.g. Faltinsen, 1978, Isaacson and Subbiah, 1991).

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The solution is expressed in terms of a set of eigenvalues α_n corresponding to each mode of sloshing. The eigenvalues correspond to $\cos(\alpha_n) = 0$, and thus are given by:

$$\alpha_n = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \qquad \text{for } n = 1, 2, 3, \ldots \tag{1}$$

The natural frequencies of each sloshing mode, denoted ω_n, may be obtained in terms of the α_n values from the equation:

$$\omega_n = \sqrt{(\alpha_n g/a)\tanh(\alpha_n h/a)} \tag{2}$$

where g is the gravitational constant. The corresponding natural periods are obtained from $T_n = 2\pi/\omega_n$.

Once α_n and ω_n are known, an available expression for the water surface elevation may be used to determine the maximum surface elevation at the reservoir walls. If the base velocity is given in complex notation as $u(t) = U\exp(-i\omega t)$, where U is the velocity amplitude, ω is the angular frequency, t is time, and $i = \sqrt{-1}$, then the water surface elevation at $x = a$ may be expressed as:

$$\eta = \frac{i\omega U a}{g}\left\{1 - \sum_{n=1}^{\infty} \frac{2}{\alpha_n^2} G_n(i\omega)\right\}\exp(-i\omega t) \tag{3}$$

where $G_n(i\omega)$ is a frequency dependent function given as:

$$G_n(i\omega) = \frac{\omega^2 + i\mu\omega}{\omega^2 + i\mu\omega - \omega_n^2} \tag{4}$$

and μ is a damping parameter which can be related to the damping ratio ζ.

It may readily be shown that Eq. 3 leads to $\eta \to 0$ in the high frequency limit. This result is significant, since it implies that there is no term analogous to the impulsive mass used in force estimates.

MODAL RESPONSE BASED ON EARTHQUAKE SPECTRUM

In the case of a base motion due to an earthquake, the motion is generally described by a specified spectral acceleration $S_a(T_n, \zeta)$, which corresponds to the maximum acceleration arising in a lightly damped single degree of freedom system of natural period T_n and damping ratio ζ and subject to a unit peak ground acceleration. The spectral acceleration for a maximum ground acceleration of 1.0g is generally approximated (e.g. National Building Code of Canada, 1985) in a simplified form:

$$S_a(T_n, \zeta) = \begin{cases} \alpha_1 & \text{for } T_n < \beta_1 \\ \alpha_2/T_n & \text{for } \beta_1 < T_n < \beta_2 \\ \alpha_3/T_n^2 & \text{for } T_n > \beta_2 \end{cases} \tag{5}$$

where α_1, α_2, α_3, β_1 and β_2 are constants which depend on the damping ratio ζ. [Subsequent editions of the code omit the long period component of Eq. 5, but this component is retained here as being of critical importance in the present context.]

Estimates based on closed-form solution

For such a motion, the maximum surface elevation η_n associated with the n-th mode of sloshing may be derived from the closed-form solution for a harmonic motion and is given as:

$$\eta_n = \left(\frac{2a}{\alpha_n^2}\right)\left(\frac{\dot{u}_M}{g}\right) S_a(T_n, \zeta) \tag{6}$$

where $\dot{u}_M$ is the maximum ground acceleration, and S_a is dimensionless with respect to g.

The traditional approach to estimating maximum forces is based on the assumption that only the first sloshing mode is significant. Using this assumption, the maximum elevation η_1 is given by:

$$\eta_1 = 0.811\,a\left(\frac{\dot{u}_M}{g}\right)S_a(T_1,\zeta) \tag{7}$$

However, for a relatively large reservoir, the spectral acceleration may be quite low at the first mode natural period so that the effect of second and higher sloshing modes may not be negligible. [This differs from the case of force predictions, since there is now no term analogous to the impulsive force component.] In this case, several modes should be considered simultaneously, and a common practice to estimating the overall maximum elevation η_M is based on the root of the squares of the maximum modal responses:

$$\eta_M = \left[\sum_{n=1}^{N} \eta_n^2\right]^{1/2} \tag{8}$$

where η_n is given by Eq. 6, and N is sufficiently large for convergence to occur.

Housner's solution

Housner (U.S. Atomic Energy Commission, 1963) has described an approximate method of solution for rectangular and circular reservoirs, in which the influence of the second and higher sloshing modes is ignored. His analysis gives the following expression for the maximum sloshing height in a rectangular reservoir:

$$\eta_H = \frac{0.527\,a}{\tanh[1.58(h/a)]\left(\dfrac{g}{\omega_1^2\theta a} - 1\right)} \tag{9}$$

where θ can be expressed as $\theta = (\dot{u}_M/g)\,S_a(T_1,\zeta)$ and ω_1 is given by Eq. 2 with α_1 taken as 1.58.

It is noted that, although Housner's solution fails for high values of θ and does not predict η_H to be proportional to base acceleration, for the practical case of low θ values the expression for η_H may be approximated by Eq. 7, but with the factor 0.811 replaced by 0.833. Thus, Housner's formulae then gives predictions which are 3% higher than the closed-form solution for first mode sloshing.

API and AWWA standards

The API (1993) and AWWA (1984) standards refer to the use of Housner's approach to estimating the maximum sloshing height. Their predictions of the maximum surface elevation are given in terms of various constants (a zone coefficient, a structure coefficient, a site amplification factor and an earthquake coefficient corresponds to the application of an assumed form of earthquake response spectrum), but in effect correspond to Eq. 7 above. The two standards are similar, except that there are slight differences in the assumed values of the constants. However, it is noted that, as with Housner's expression, these predictions are based on the assumption that the second and higher sloshing modes may be ignored.

RESULTS AND DISCUSSION

Harmonic Excitation

The solution for a harmonic excitation, Eq. 3, indicates that the dimensionless amplitude of the free surface elevation $\hat{\eta}$ is a function of the dimensionless frequency ω^2a/g, the relative depth h/a, and the damping ratio ζ. Figure 2 shows the dimensionless amplitude of the free surface elevation at the wall x = a as a function of ω^2a/g for various values of damping ratio ζ and for h/a = 1. The figure clearly shows the large elevation amplitudes at the lower sloshing modes corresponding to $\omega_n^2\,a/g$ = 1.57, 4.85 and 8.10 for n = 1, 2, 3.

Earthquake Excitation

For the case of earthquake motions, it is of interest to compare the complete closed-form solution with that for first mode sloshing and with Housner's solution. The comparison between Housner's solution, denoted η_H, and the closed-form solution for first mode sloshing, η_1, is conveniently presented as η_H/η_1. This depends on the reservoir size a, the relative depth h/a, the damping ratio ζ, and the maximum ground acceleration $\dot{u}_M/g$. In all cases a damping ratio $\zeta = 0.005$ is assumed. Figure 3 shows this ratio for various values of h/a and for $\dot{u}_M/g = 0.1$ (Fig. 3a), and for various values of $\dot{u}_M/g$ and for h/a = 1.0 (Fig. 3b). The figure indicates how Housner's solution becomes inaccurate relative to the first order solution for smaller reservoirs and for higher values of base acceleration. Figure 4 provides a comparison of the full closed-form solution and the closed-form solution for first mode sloshing. The ratio η_M/η_1 is shown as a function of reservoir size a for various values of h/a and for $\dot{u}_M/g = 0.1$. As expected, the figure indicates that the first-mode result underpredicts the results of the complete solution most significantly for larger reservoir sizes. This is most pronounced for relatively shallow large reservoirs. In particular, for the case h/a = 0.2 and for reservoir lengths 2a > 60 m, the maximum elevation based on the complete solution is more than 50% higher than that based on first mode sloshing.

Additional Factors

Irregular planform. When the reservoir has a circular planform, the identical approach can be used, except that the α_n values are different (α_n = 1.84, 5.33, 8.54 for n = 1, 2, 3). When a reservoir has an irregular planform that cannot readily be approximated as circular or rectangular, the numerical method described by Isaacson and Ryu (1998a) may be adopted. The approach used is based on an eigenfunction expansion of the velocity potential with respect to the vertical direction combined with a two-dimensional boundary element method with respect to the horizontal plane, with the planform of the reservoir discretized into a number of short segments.

Effects of direction. In certain cases, it may be necessary to consider a reservoir motion which is uni-directional but not parallel to a pair of sides. A direction of motion which is oblique can be analyzed by an appropriate superposition of two component motions parallel to the two pairs of sides. This enables the known closed-form solution to be extended to the case of an oblique direction. Results for the case of an oblique direction given by Isaacson and Ryu (1998b) indicate that for earthquake motion, a direction of motion parallel to the shorter pair of sides always gives the highest loads and surface elevations. Therefore, the common assumption that the earthquake motion acts in a direction parallel to either pair of sides is appropriate.

Damping. Energy dissipation may occur on account of viscous effects associated with boundary layers on the reservoir walls, particularly for boundaries which are rough or contain protrusions; flow separation effects as the fluid oscillates past baffles or other obstacles in the reservoir; and free surface effects associated with breaking waves. As already indicated, energy dissipation may be accounted for in the complete solution, provided that a suitable damping ratio is assumed. The damping ratio may be estimated from an assessment of the energy dissipation mechanisms that are predominant. Approaches to estimating hydrodynamic damping due to baffles or perforated bulkheads and wave breaking have been indicated by Isaacson and Subbiah (1991). Unless more detailed information is available, a damping ratio of 0.005 appears to be reasonable for general application.

Example

Using the procedure outlined above, a sample set of calculations has been carried out for a reservoir which is 300 m long, 150 m wide, and has a water depth of 10 m. For such a condition the natural periods of the lowest modes for longitudinal sloshing are 60.54 s, 20.47 s, and 12.61 s. For a maximum ground acceleration $\dot{u}_M = 0.2g$, and a damping ratio $\zeta = 0.005$, the estimates of the maximum elevation are $\eta_M = 0.156$ m, $\eta_1 = 0.064$ m and $\eta_H = 0.066$ m. Therefore, the maximum sloshing height should be taken as 0.156 m. The above results confirm that the estimation of maximum surface elevation in a large reservoir requires the influence of higher sloshing modes to be taken into account. As expected, prediction based on the expression by Housner are 3% higher than those for first mode sloshing given by Eq. 7. It is of interest to contrast this to the case of the force calculations. The maximum force based on the complete solution is 3.64 MN, whereas the maximum force based on first sloshing mode is 3.62 MN. This small difference arises because of the relatively large influence of the impulsive mass, and is in significant contrast to the much larger differences in free surface elevation predictions.

SUMMARY AND CONCLUSIONS

The present paper provides an overview of approaches used to estimate the maximum water surface elevation in reservoirs due to earthquake-induced sloshing. These correspond to predictions based on the complete closed-form solution to the linearized boundary value problem, the corresponding solution for first mode sloshing only, and the early solution of Housner. These three approaches are compared. It is shown that the reliable estimation of the maximum surface elevation in a large reservoir requires the influence of higher sloshing modes to be taken into account. And, as expected, Housner's predictions correspond closely to those based on the closed-form solution for first mode excitation, but give predictions which are slightly higher. A brief indication is also provided of various additional factors which may require consideration, including the treatment of reservoirs with complex planforms; the influence of the direction of base motion; and the estimation of hydrodynamic damping.

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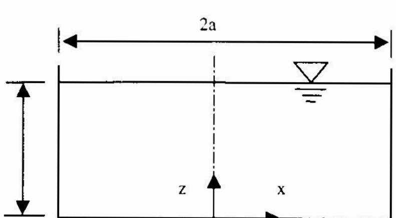

Figure 1. Definition sketch of a rectangular reservoir.

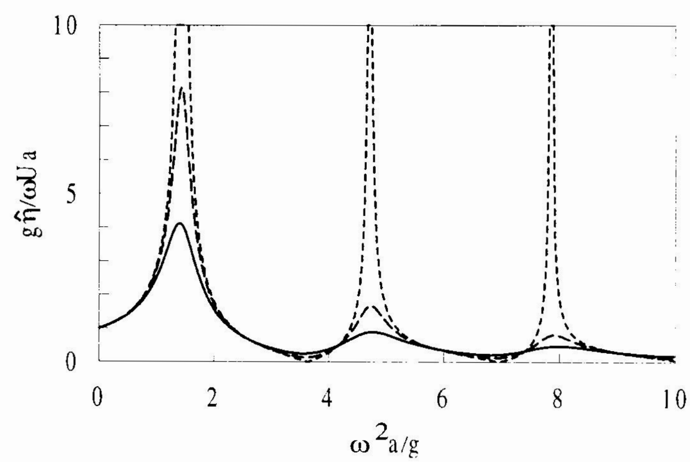

Figure 2. Dimensionless amplitude of free surface elevation at tank wall as a function of $\omega^2 a/g$ for h/a = 1. -----, $\zeta = 0.00$, — — —, $\zeta = 0.05$, ——, $\zeta = 0.10$.

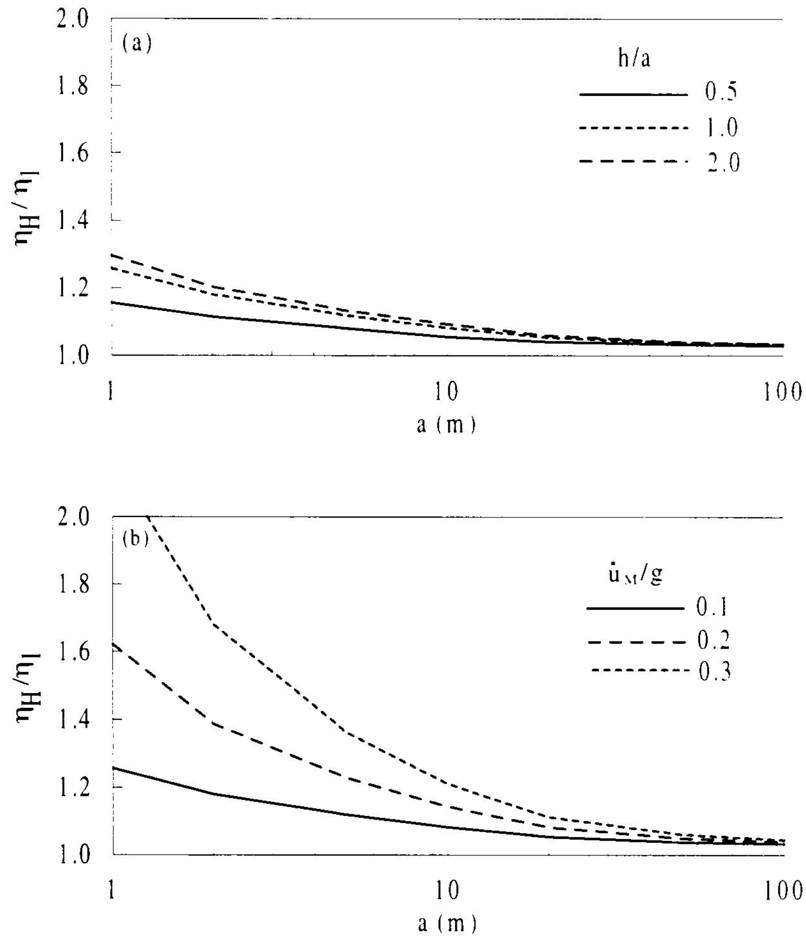

Figure 3. Comparison of elevations based on Housner's solution and the closed-form solution for first mode sloshing. (a) $\dot{u}_M/g=0.1$ and various h/a values; (b) h/a = 1.0 and various $\dot{u}_M/g$ values.

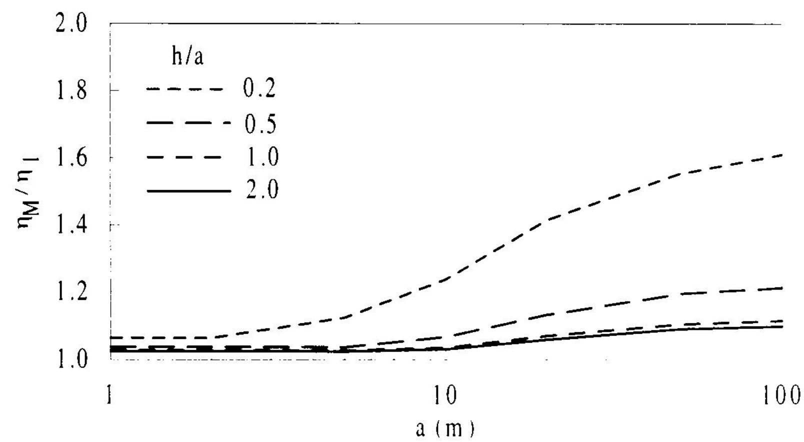

Figure 4. Comparison of elevations based on the complete closed-form solution and the first mode solution for $\dot{u}_M/g=0.1$ and various h/a values.